Probability distributions of peaks, troughs and heights of wind waves measured in the black sea coastal zone

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Abstract

The present paper examines the adequacy of different probability density functions to describe the peaks, troughs and peak-to-trough excursions of wind waves measured in the coastal zone of the Bulgarian part of the Black sea. For that purpose various theories for non-Gaussian random process are applied. Some theories depend on the mean, variance and coefficient of skewness $\gamma_3$ of the process. Others also take the coefficient of kurtosis $\gamma_4$ into consideration. The analyzed field data are gathered in the coastal zone of the Bulgarian part of the Black sea with depth decreasing from 18 to 1.3 m. The measurements are carried out simultaneously for 11 points with time series of 20 min duration. The coefficients of skewness and kurtosis in those time series are expressed as functions of depth and spectral peak frequency. Analogous dependencies on depth of other parameters are also found. As a result of the investigation it is concluded that the probabilities of occurrence of large wave crests and heights are underpredicted by all of the theories considered.

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1. Introduction

Records of the deviation of the free surface from the mean water level constitute the basic information on shape of water waves and they show that surface waves are both irregular and skewed. This type of records has been extensively used to validate various theories of linear and non-linear waves. Due to non-linearity the crests are higher and sharper and the troughs are shallower and flatter as opposed to the linear case where they are symmetrical. As a result of non-linear interactions very high waves can be encountered in deep water as well as in shallow water (Pelinovsky and Kharif, 2003).

Longuet-Higgins (1963) modelled the wave profile considering for the first time the non-linear interactions between wave modes. He showed that under assumption of weak non-linearity the surface profile...
could be described by means of Gram–Charlier series, type A in the form of Edgeworth. The applicability of the Gram–Charlier representation was confirmed by many authors, e.g. Huang and Long (1980) for the deep-water case and Bitner (1980) for the shallow water case. The major problem of the resulting probability density function for the surface elevation is that it becomes negative over some range of its values. That drawback was overcome by Srokosz (1998) who derived a new non-linear distribution from a Pearson system of distributions. Nevertheless, neither the theory of Longuet–Higgins, nor that of Srokosz can be further used for predicting the wave crests distribution.

Another probability model is that of Ochi and Ahn (1994). The probability function of ordinates is derived in a closed form following the Kac–Siegert solution of Volterra’s stochastic series expansion applicable to the response of a non-linear system. The response is given as the sum of a non-Gaussian random process and a sine wave.

For practical purposes it is more appropriate to use the statistical distributions of wave parameters, instead of those of ordinates. In deep water the wave amplitude distribution is given theoretically by means of the Rayleigh law (Rice, 1945, Longuet-Higgins, 1952). The additional assumption that the wave height is equal to twice the wave amplitude makes the Rayleigh distribution applicable to wave heights. Several studies confirmed the validity of the Rayleigh law in deep water (Earle, 1975). Nevertheless, Haring et al. (1976), Forristall (1978) and Guedes Soares and Carvalho (2003), among others, found some discrepancies with field data. The inability of the Rayleigh law to describe real waves is due to the fact that it does not take into account the non-linear interactions between spectral wave components responsible for the vertical asymmetry in the wave profile, which was discussed recently by Guedes Soares et al. (2004).

Longuet-Higgins (1980, 1983), and Tayfun (1980) examined the effects of non-linearity and finite bandwidth on the distribution of wave heights and amplitudes. Tayfun (1980) showed that even if the surface displacement was non-Gaussian, wave heights followed the Rayleigh law but crests were non-Rayleigh, unlike their linear counterparts. The concept was further applied in Tayfun (1983, 1984). The wave height distribution, considered as the sum of two independent extremes separated by one-half of the mean zero-up-crossing wave period, was derived by Tayfun (1981, 1990).

Several studies on the probability distribution of wave heights were carried out in finite water depth as reviewed for example by Guedes Soares (2003). In contrast to deep-water conditions, there is no theoretical distribution that can be generally applied. This is due to the specific properties of wind waves, which are subjected to shoaling, transformation, non-linear wave–wave interactions and breaking during the process of shoreward propagation. Bitner (1980) developed an approximate distribution for heights following the Hermite polynomials concept for the surface elevation. The probability density functions of crests and troughs of Tayfun (1980) were derived considering the wave amplitude as the sum (or difference for the case of troughs, respectively) of Gaussian amplitude and its squared quantity. Both approaches, however, assume the probability distributions of crests and troughs to be the same.

A different approach was presented by Arhan and Plaisted (1981). Introducing a specific parameter, they demonstrated the differences between statistics of crests and troughs. Ochi (1998) derived analytically the probability density functions of crests and troughs applying the concept of non-linear system response function. He only took into consideration the coefficient of skewness, i.e., non-linear interactions of second order. Nonlinear second order effects were also presented in the distribution functions of Forristall (2000), Al-Humoud et al. (2002) and Wist et al. (2002).

Mori and Yasuda (2002) studied the effects of higher order non-linear interactions by applying also the coefficient of kurtosis in addition to the coefficient of skewness. They pointed out that the third order non-linearities have dominant effect on the wave height distribution. An account of how these coefficients depend on the intensity of the sea states was considered in Guedes Soares et al. (2003).

For the conditions of the Bulgarian Black Sea coast the applicability of the empirical distribution of Gлушовский (1966) was checked by Belberov et al. (1980). Although widely applied it did not yield good results (Belberov et al., 1990).
In more recent work the distributions of Longuet-Higgins (1980, 1983) and Tayfun (1990) were investigated in order to trace the transformation of wave heights distribution in shallow water (Cherneva and Velcheva, 1995). It has been shown that the theory of Longuet-Higgins (1983) is inconsistent under those conditions since it considers the wave height as two times the wave amplitude, which is not appropriate in shallow water.

The goal of the present paper is to analyse the applicability of the distributions of Ochi (1998), Al-Humoud et al. (2002) and Mori and Yasuda (2002) to the case of non-linear, non-Gaussian waves measured in the Bulgarian coastal zone.

The basic methods applied in the analysis are presented in Section 2. They can be separated into two groups depending on the order of non-linearity taken into consideration. The first group takes into account second order non-linear interactions expressed by means of the coefficient of skewness (Ochi, 1998, Al-Humoud et al., 2002) and the second group considers the third order non-linearities applying the coefficient of kurtosis, except for the coefficient of skewness (Mori and Yasuda, 2002). It must be noted that the latter approach was derived following some typical assumptions for the deep-water case that will be discussed in detail in the following.

In Section 3 field data and their analysis are described. Field data from different depths in the western part of the Bulgarian Black Sea coast are fitted with the distributions of Ochi (1998), Al-Humoud et al. (2002) and Mori and Yasuda (2002).

The variations of the coefficient of skewness \( \gamma_3/k_3^{3/2} \) and coefficient of kurtosis \( \gamma_4/k_4^{2} \) with \( \omega_h = \omega_p \sqrt{h/g} \) (\( k \) stand for cumulants of \( i \)-th order; \( \omega_p \) is the spectral peak frequency; \( h \) is the water depth and \( g \) is the gravity acceleration) are studied and some empirical relations are found. Thus, it becomes possible to follow how the distributions of wave parameters are changing with depth. Relationships between the parameters \( a, \mu^* \) and variance \( \sigma^* \) of the distribution of Ochi (1998) and parameter \( \omega_h \) are also presented here.

Subsequently, the Ursell parameter \( Ur = (2\pi)^{2} \frac{\bar{a} \sigma_{x}}{(\kappa_{g}h)} \) has been considered with relation to \( \omega_h \), given \( \bar{a} \) is the mean wave height, expressed as half of the mean wave height \( \bar{H} \), i.e. \( \bar{a} = \bar{H}/2 \); \( \kappa_{g} \) is the wave number, corresponding to the spectral peak, calculated from the linear dispersion relationship. The parameter \( Ur \) is used to conclude on the prevalence of non-linearity or dispersion in the wave field during the wave propagation to the shore (Nekrasov and Pelinovsky, 1992).

The results are discussed in Section 4.

2. Models of probability distributions

2.1. Second order model of Ochi

The distributions of wave crests, troughs and heights proposed by Ochi (1998) are based on the concept of non-linear system response function expressed in the form of Volterra’s second order functional series. Kac and Siegert (1947) derived the following solution for the response of the system

\[
y(t) = \sum_{j=1}^{n} \left( \beta_j Z_j + \lambda_j Z_j^2 \right)
\]

where \( y(t) \) is the deviation from the mean value, expressed as a summation of a normal random process and its squared quantity, where for each fixed value of \( t \), \( \{Z_j(t)\}_{j=1,\ldots,2N} \) is a set of independent zero-mean Gaussian variables. The coefficients \( \beta_j \) and \( \lambda_j \) are derived from the second order potential theory. Instead of Eq. (1) Ochi and Ahn (1994) proposed a new representation of the non-linear process in order to derive the probability density function of the surface elevation in a closed form. The random process is given as a function of a single random variable. Then \( y(t) \) is written as

\[
y(t) = U + aU^2
\]

where \( a \) is a constant, \( U \) is a normal variable with mean \( \mu^* \) and variance \( \sigma^* \). The parameters \( a, \mu^* \) and \( \sigma^* \) are obtained by solving the following non-linear system of equations

\[
\begin{align*}
|a\sigma_{x}^{2} + a\mu_{x}^{2} &+ \mu_{x}^{2} = k_1 = 0 \nonumber \\
\sigma_{x}^{2} - 2a^{2}\sigma_{x}^{4} &+ 4a^{2}\sigma_{x}^{2} = k_2 \nonumber \\
2a^{4}\sigma_{x}^{2} &\left(3 - 8a^{2}\sigma_{x}^{2}\right) = k_3
\end{align*}
\]

where \( k_{l=1,2,3} \) represents cumulants of order \( l \).
The probability density function of ordinates is obtained as
\[
f(y) = \frac{1}{\sqrt{2\pi\sigma_y}} \exp \left\{ -\frac{1}{2(\gamma\sigma_y)^2} (1 - \gamma y) \right\}
\]
\[
\gamma = \begin{cases} 
1.28, & y \geq 0 \\
3.00, & y < 0
\end{cases}
\]

Subsequently, the probability density function of peaks and troughs is expressed as equivalent to the density function of the envelope of a random process, given as sum of a narrow-band Gaussian process and a sine wave. Two expressions are analytically derived for the probability density functions of crests \( f(d) \) and troughs \( f(\eta) \), respectively. The expressions are similar to the solution of Rice (1945). The exact forms of the distributions are given as follows
\[
f(d) = \frac{d}{s_1^2} \exp \left\{ -\frac{1}{2s_1^2} \left( c_1^2 + d^2 \right) \right\} I_0 \left( c_1 \frac{d}{s_1} \right), 0 \leq d < \infty
\]
\[
s_1^2 = \frac{\sigma_d^2}{1 + \lambda_1 \mu_1}
\]
\[
c_1 = \frac{\sqrt{2} (\lambda_1 \sigma_1^2 - \mu_1)}{1 + \lambda_1 \mu_1}
\]
\[
\sigma_1^2 = \frac{2}{\lambda_1^2} \left[ \frac{1}{2} (\lambda_1 \sigma_1^2) + \sqrt{\frac{2}{\pi}} (\lambda_1 \sigma_1)^3 + \frac{3}{2} (\lambda_1 \sigma_1)^4 \right]
\]

where \( s_1^2 \) and \( c_1 \) stand for the variance of the Gaussian process and amplitude of the sine wave, respectively; \( \lambda_1 = a \gamma, \gamma = 1.28; \sigma_1^2 \) is the standard deviation of the upper envelope process, denoted by \( \xi(t) \) and \( I_0(\cdot) \) is the modified Bessel function of first kind and zero order. The parameters \( \alpha, \mu_1, \sigma_1 \) of the distribution are calculated from (3).

An analogous expression is obtained for the distribution of troughs, given that \( s_1^2 \) and \( c_1 \) are substituted by \( s_2^2 \) and \( c_2 \), respectively, and \( \sigma_1^2 \) is replaced by \( \sigma_2^2 \), where \( \sigma_2^2 = 2E[y^2] - \sigma_1^2 \) and \( \lambda_2 = a \gamma, \gamma = 3.00; E[y^2] \) stands for the variance of the process. The distribution has the form
\[
f(\eta) = \frac{\eta}{s_2^2} \exp \left\{ -\frac{1}{2s_2^2} \left( c_2^2 + \eta^2 \right) \right\} I_0 \left( c_2 \frac{\eta}{s_2} \right), 0 \leq \eta < \infty.
\]

The modified Bessel function \( I_0(z) \) can be approximately expressed in exponential form when the argument value is less than about 2.5–3 (see Fig. 2 in Ochi (1998)), i.e.
\[
I_0(z) \approx \exp \left( \frac{z^2}{5} \right)
\]

Then, Eqs. (6) and (10) reduce to
\[
f(d) = \frac{1}{s_1^2} \left( 1 - 2c_1^2 \right) \xi \exp \left[ -\frac{1}{2s_1^2} \left( 1 - 2c_1^2 \right) \xi^2 \right], 0 < \xi < \infty
\]
\[
f(\eta) = \frac{1}{s_2^2} \left( 1 - 2c_2^2 \right) \eta \exp \left[ -\frac{1}{2s_2^2} \left( 1 - 2c_2^2 \right) \eta^2 \right], 0 < \eta < \infty
\]

Expressions (12) and (13) represent themselves the Rayleigh probability function with parameters \( R_{t1,2} \) indicating the non-Gaussian characteristics of the process
\[
R_{t1,2} = \left( \frac{2s_1^2}{R_1 + R_2} \right)^2 \left[ 1 - \left( \frac{2s_1^2}{5s_1^2} \right) \right]
\]
where the index 1 is related to wave crests and the index 2—to wave troughs, respectively.

The peak-to-trough excursions \( \xi \) are considered to be the sum of two independent random variables \( \xi \) and \( \eta \). Subsequently, the probability density function of \( \xi \), denoted as \( f(\xi) \) is given as a convolution integral of the two probability functions \( f(\xi) \) and \( f(\eta) \). Assuming the approximate solutions (12) and (13), \( f(\xi) \) is obtained as follows
\[
f(\xi) = \frac{2\xi}{(R_1 + R_2)^2} \left( R_1 \exp \left( -\frac{\xi^2}{R_1} \right) + R_2 \exp \left( -\frac{\xi^2}{R_2} \right) \right) + \frac{2\sqrt{\pi}}{R_1(R_1 + R_2)} \exp \left( -\frac{\xi^2}{R_1 + R_2} \right) \sqrt{\frac{R_1 R_2}{R_1 + R_2}}
\]
\[
\times \left( \frac{2\xi^2}{R_1 + R_2} - 1 \right) \left\{ \Phi \left[ \sqrt{\frac{2R_2}{R_1(R_1 + R_2)^\xi}} \right] - \Phi \left[ -\sqrt{\frac{2R_1}{R_2(R_1 + R_2)^\xi}} \right] \right\}, 0 < \xi < \infty
\]
where \( \Phi[\ldots] \) is the cumulative distribution function of the standardized normal process.

### 2.2. Second order model of Al-Humoud, Tayfun and Askar

The non-linear second-order model of Al-Humoud et al. (2002) is based on the truncated Gram–Charlier series description of the surface profile. No restrictions regarding the directional or spectral properties of waves are imposed on the solution.

Let \( Y(t) \) and \( \tilde{Y}(t) \) be the non-linear surface elevation and its Hilbert transform, respectively.

\[
Y(t) = A(t) \cos \phi(t) 
\]

\[
W(t) = Y(t) + i\tilde{Y}(t) = A(t) \exp(i\phi(t)) 
\]

\[
A(t) = \sqrt{Y^2(t) + \tilde{Y}^2(t)} 
\]

\[
\phi(t) = \tanh \left( \frac{\tilde{Y}(t)}{Y(t)} \right) 
\]

where \( A \) stands for the random amplitude and \( \phi(t) \) for the random phase, uniformly distributed in the interval \([0,2\pi]\). The probability density functions of the normalized random amplitudes over the crest and trough segments are given by the following formula

\[
p^\pm(b) = \frac{1 \pm c_1 \gamma_3 b(b^2 - 2)}{1 + c_0 \gamma_3} p(b) 
\]

\[
b = \frac{A}{A_{rms}}; \quad A_{rms} = (2m_0)^{1/2} 
\]

with \( \gamma_3 \) being the coefficient of skewness for \( Y(t) \); \( m_0 \)—the zero-order spectral moment; \( c_0 = \frac{1}{\sqrt{2\pi}} \) and \( c_1 = \frac{2\sqrt{2}}{\sqrt{\pi}} \)—the corrections, required in the expressions for \( p^\pm \), given earlier by Tayfun (1994), where \( p^+ \) and \( p^- \) are the distributions of crests and troughs, respectively. When \( \gamma_3 \to 0 \), \( p^\pm(b) \to p(b) \) where \( p(b) = 2b\exp(-b^2) \) is the Rayleigh distribution as should be expected for the linear case.

### 2.3. Model of Mori and Yasuda

Mori and Yasuda (2002) developed a non-Gaussian model for wave and crest heights distribution on the base of Gram–Charlier series representation of the surface elevation. The advantage of the model is that it also considers the third order non-linearity by means of the coefficient of kurtosis. It must be noted also that the model was empirically derived and subsequently verified for deep-water conditions. The process is assumed to be weakly non-linear with narrow frequency band spectrum. The wave heights are presented as twice the wave amplitude, i.e. the crests and troughs are considered symmetrical. Additionally, the waves are considered to be unidirectional. The notation in Eqs. (16) (17) (18) (19) is also adopted here, in order to describe the wave process and its amplitude. Although the conditions in the coastal zone differ substantially from deep water ones, the present study aims to check the model applicability to shallow water.

The probability density functions of wave amplitudes and heights, given the wave height as twice the wave amplitude, are derived by means of change of the standardized variables \( Y', \tilde{Y}' \) to \( a, \phi \) in the joint probability density function of \( (Y', \tilde{Y}') \) where

\[
Y' = \frac{Y}{Y_{rms}}; \quad \tilde{Y}' = \frac{\tilde{Y}}{Y_{rms}}; \quad Y_{rms} = E[Y^2] 
\]

and \( b \) is the normalized random amplitude in accordance with Eq. (21).

The local wave amplitude distribution is given then by

\[
p(b)db = \frac{b}{8} \exp \left( -\frac{b^2}{2} \right) 
\]

\[
\times \left[ 1 + \sum_{i=1}^{2} \alpha_{4,i} A_{4,i}(b) + \sum_{i=1}^{3} \alpha_{6,i} A_{6,i}(b) \right] db 
\]

(23)

where \( \alpha_{4,i} \) and \( \alpha_{6,i} \) are coefficients depending on the third and fourth central moments (\( \mu_3 \) and \( \mu_4 \)). These coefficients together with the polynomials \( A_{ij} \) given as functions of the normalized amplitude \( b \) are presented in detail in the text and in the Appendix in Mori and Yasuda (2002).

Subsequently, the following expression for the probability density function of heights \( H' \) is obtained

\[
p(H')dH' = \frac{H'}{4} \exp \left( -\frac{H'^2}{8} \right) \left[ 1 + \sum_{ij} \beta_{ij} B_{ij}(H') \right] dH' 
\]

(24)
where $H' = H/Y_{rms}$, $H' = 2b$ and $\beta_{ij}$, $B_{ij}$ are also presented in Mori and Yasuda (2002).

### 3. Analysis of field data

The experimental wave data were collected at the research station “Shkorpilovtsi” during the international experiment “WAVE’90”. The station is located at about 40 km southward from Varna and it is suitable for observations of wind waves, generated by winds from all eastern sectors. These are the directions, critical for the formation of wind waves along the Bulgarian Black Sea coast. The most characteristic features of the coastal zone in “Shkorpilovtsi” are the mild bottom slope, rectilinear shoreline and almost parallel isobaths. Within the depth of 0–10 m the mean bottom slope is 0.025. From depth of 10–20 m it is 0.006 and the effect of wind wave reflection can be neglected, because underwater bars are practically not encountered here (Fig. 1).

The data were taken during the initial stages of two storms: from November 30th to December 1st and from December 2nd to December 4th. The measurements of sea surface elevation were performed with string recorders of resistant type at eleven fixed points. Nine of the gauges are mounted on a pier, 250 m long, orientated perpendicularly to the shore and the others are mounted on offshore towers. The location of the gauges, as well as information on the water depth and offshore distance at the points is given in Fig. 1 and Table 1.

The measurements from the experiment “WAVE’90” were simultaneously carried out at every 3 h with 20-min duration at sampling frequency of 0.160 s for the wave recorders installed on the pier (gauges 1–9) and at 0.165 s for the gauges 10 and 11 located on the towers. Thus, the zero-up-crossing wave heights counted in each record is on the average 120–200. Although the number of broken waves in the records is increasing shoreward, all data are processed in the same manner, without taking into consideration breaking or broken waves. The experimental data are checked for accuracy of the recording. False data are excluded from further analysis and then statistical and spectral characteristics are calculated.

A correction of the linear change of mean water level proposed by Goda (2000) is first applied to the original records of ordinates. Then the data are standardized by subtracting the mean value and dividing by the standard deviation, calculated as the second order cumulant of the wave process. Thus, the new process has zero mean and unit variance. The waves are defined using the down-crossing method. The maximum positive extrema for a wave is set to determine the wave crest and the maximum of the negative extrema—the wave trough.

Fig. 2 is representative of the changes with depth of the mean wave height ($\bar{H}$) and mean wave period ($T_{02}$). The mean wave period is calculated trough the zero and second order spectral moments, $T_{02} = \sqrt{m_0/m_2}$. The results for four different terms on December 2nd are shown. It is clearly seen that the mean wave height starts to increase at 225 m offshore (point 9), reaches its maximum at 185 m (point 7) and then decreases (Fig. 2a). The trend remains for the whole period of the two storms (Cherneva and Vel-
During the development of the storms, the peak of the mean wave height becomes more pronounced. Fig. 2b generally shows that $T_{02}$ is almost constant till 225 m and then decreases. Hence, Fig. 2 indicates not only the spatial variation of the considered wave characteristics, but also their time variations with the development of the storms: an increase with time is observed.

A portion of the time history, pertaining to the day with the severest storm conditions, showing typical asymmetric look of non-linear, non-Gaussian random process is presented in Fig. 3. The mean wave height, calculated from the observed wave heights in the record is 0.81 and the coefficient of skewness is 1.0505.

There are two parameters that can be used to describe the transformation of wave elements’ distribution in the coastal zone—the Ursell parameter $U_r$ and $\omega_h$. Their calculated values are put together in Fig. 4 and subsequently a power dependence is found, expressed by the following equation

$$U_r = 0.1374 \omega_h^{-3.5510}.$$

The experimental data denoted with different symbols refer to November 30th and December 2nd. The Ursell parameter shows the ratio between non-linearity and dispersion in shallow water wave models. The value $U_r=4$ refers to the transition between these two processes, as was discussed in Nekrasov and Pelinovsky (1992). In the present case the critical value
corresponds to $\omega_h \approx 0.4$. Consequently, if $Ur \gg 4$ and $\omega_h \ll 0.4$ the non-linear effects are significant as compared to wave dispersion. On the contrary, if $Ur \ll 4$ and $\omega_h \gg 0.4$, the dispersion prevails. In deep water, the values of $Ur$ are small. As waves propagate to the shore $Ur$ increases, the front crest side of waves becomes steeper and finally they break. Furthermore, the parameter $\omega_h$ is used instead of $Ur$ because $\omega_h$ can be more easily obtained, while the definition of $Ur$ implies knowledge on wave length. Here, a compromise is made calculating the wave length through the wave number and linear dispersion relationship. Considering Fig. 4, the values of $\omega_h$ can be divided into three intervals in a shoreward dire-

c tion: $1.1 < \omega_h < 1.8$, $0.4 < \omega_h < 1.1$ and $\omega_h < 0.4$. The deepest region refers to prevailing dispersion. Then, in the second interval the combined effect of non-linearity and dispersion comes into force. The non-linearity rapidly increases and becomes significant in the third region, due to the fact that shoreward propagating waves are getting longer. The long wave theory assumes that there is no dispersion and the phase speed depends only on local water depth. As a whole, most of the experimental data are located in the interval $0.4 < \omega_h < 1.1$.

The wind wave profile is investigated through the non-dimensional parameter $H_{cr}/H_{tr}$, where $H_{cr}$ and $H_{tr}$ represent the mean crest height and mean trough depth, respectively. The variations of $H_{cr}/H_{tr}$ with $\gamma_3$ and $\gamma_4$ are given in Fig. 5. Three cases of November 30th and one case on December 2nd are used. The corresponding designation is given opposite each of the symbols in the legends. It is found that $H_{cr}/H_{tr}$ depends linearly on $\gamma_3$ and $\gamma_4$. The correlation is stronger in Fig. 5a with coefficient $R = 0.9402$. It implies that the vertical asymmetry is basically due to the second order non-linearity. This is a good reason to use the approximations, proposed by Ochi (1998) and Al-Humoud et al. (2002) that depend only on $\gamma_3$.

At the same time, the vertical wave asymmetry becomes more pronounced—higher crests and shallower troughs—as the water depth decreases (Fig. 6).

As the computation of the probability functions depends critically on the coefficient of skewness
(Ochi, 1998) and also on the coefficient of kurtosis (Mori and Yasuda, 2002), an accurate estimation of the parameters $\gamma_3$ and $\gamma_4$ is required to fit these two types of distribution. The influence of the record length on the estimation of the coefficients of skewness and kurtosis is investigated and the results are given in Fig. 7.

The estimated values of the two parameters for four simultaneous measurements, taken on November 30th at 22 h at different depths (1.30, 2.60, 3.80 and 4.20 m), are plotted versus the number of measured ordinates. The coefficient of skewness is changing moderately while kurtosis is statistically quite unstable as expected for higher order cumulants. It follows from the figures that the estimates of the two parameters become stable after approximately 3200 ordinates (nearly 9 min) and this must be taken into consideration when evaluating both $\gamma_3$ and $\gamma_4$.

Results on the variation of $\gamma_3$ and $\gamma_4$ with $\omega_h = \omega_p \sqrt{h/g}$ are given in Fig. 8. The deep-water case corresponds to $\omega_h \approx 2$ (Kostichkova et al., 1990). The scatter diagrams (Fig. 8) are created using data from all records obtained during the two storms. They are approximated with exponential curves with the following equations

\begin{align}
\gamma_3 &= 2.9424\exp(-2.8473\omega_h) \\
\gamma_4 &= 2.7731\exp(-2.4857\omega_h)
\end{align}

It follows from Eqs. (26) and (27) that when $\omega_h$ increases, $\gamma_3$ and $\gamma_4$ asymptotically tend to 0. Eq. (26) is used to obtain the value of $\gamma_3$, given a certain value of $\omega_h$. Subsequently, $\gamma_3$ is used as an input parameter to solve the non-linear system of Eq. (3), in order to find the probability functions of wave parameters (Ochi, 1998). The results are presented as Approximation II in Fig. 9a,b, and c. Approximation I represents the fitting curve to the experimental values of the parameters $a$, $\mu_*$ and $\sigma_*$ drawn with respect to $\omega_h$. The approximation lines I have the following exponential form

\begin{align}
a &= 0.4697\exp(-2.8346\omega_h) \\
\mu_* &= -0.0952\exp(-3.3986\omega_h) \\
\sigma_* &= 0.0970\exp(-3.4128\omega_h) + 1
\end{align}

Fig. 7. Sampling variability of the coefficient of skewness $\gamma_3$ and coefficient of kurtosis $\gamma_4$ on November 30th at 22 h for different points in the coastal zone.
The values of $a$, $l^*$ and $r^*$ are obtained for a certain $x_h$ using expressions (28) (29) (30) and the probability densities are then evaluated. The discrepancy between the two methods of evaluating the probability density is very small and, in fact negligible. A slight disagreement between the approximations of the parameters $l^*$ and $l^*$ is shown in Fig. 9b and c, but data are well described as a whole by both curves. The results for the parameters $a$, $l^*$ and $r^*$ confirm the tendency, predicted by Ochi (1998).

The skewed nature of the surface profile is expressed by means of the probability distribution of surface ordinates, given by Ochi and Ahn (1994). It is further compared with the experimental distribution and the Gaussian distribution (Fig. 10). The depth is decreasing from 18 (Fig. 10a) to 1.3 m (Fig. 10c) while at the same time the coefficient of skewness is increasing from 0.1036 to 0.8372, respectively. It is seen that the distribution of Ochi for the local surface elevation approximates field observations very well and in contrast to the distribution of Gram–Charlier it has the advantage of not taking negative values.

The three cases represent a shift to the left as compared to the Gaussian distribution. The vertical asymmetry of the wave profile, manifested in higher crests and shallower troughs, causes a change in the distribution of ordinates: the probability of large positive ordinates of the normalized process increases while the probability of small negative ordinates decreases. The case, represented in Fig. 10a, refers to almost deep water where $\gamma_3 \to 0$. It is seen that in this case there is a negligible deviation in the prediction of linear theory and the theory, assuming second order of approximation.

Comparison between the theoretical distributions applicable to wave crests and the experimental data are shown in Fig. 11. The triangles denote the observations and the solid line indicates the Rayleigh distribution. The parameter $a_h$ is also included in the figures and the transformation of distributions in shallow water is considered assuming the values of $a_h$.

It is seen that the Rayleigh distribution systematically underpredicts the observed values, except for the case in Fig. 11b, where the Rayleigh distribution shows the maximum probability of exceedance for a certain wave crest, due to the negative values of the coefficients of skewness and kurtosis. The observed underprediction can be explained with the assumption of equality between wave crest and trough. All of the considered theoretical distributions with non-linear corrections show enhanced probability, as regards the high wave crests in comparison with the Rayleigh theory. The lowest probability of exceedance is predicted by the distribution of Mori and Yasuda (2002). Although it accounts for the coefficient of kurtosis in the occurrence of large wave crests and heights it is not able to describe the considered experimental data. To explain that, it suffices here to recall that this distribution implies deep-water conditions and narrow spectrum. The distribution of Al-Humoud et al. (2002) best depicts the real data in the range of small wave crests. However, the experimental results usually show greater probability for high crest tail of the
distribution. At the deep-water point (Fig. 11a) the distributions are getting nearer to each other and to the histogram of field data. However, it was established that for our data the depth is not a determinant factor for the form of the certain distribution.

Similar comparison is done for the wave troughs (Fig. 12). Data are designated by squares. In contrast to the case of wave crests, the theoretical probability of exceedance of troughs given by the Rayleigh law is higher than observations. The rest of the distributions, except for that of Mori and Yasuda (2002), decrease the probability of occurrence for a certain trough value. This fact is again consistent with the non-linear asymmetric profile of the wind waves. The experimental data demonstrate that the distribution of wave troughs is best fitted by Ochi’s approximation (Ochi, 1998), as shown in Fig. 12c,d and e. Some deviations occur either when $\gamma_3$ takes negative values or when $\gamma_3$ is large. For $\gamma_3 < 0$ or when both $\gamma_3$ and $\gamma_4$ are negative, the wave troughs are well described by the theory of Al-Humoud et al. (2002) (Fig. 12a). Usually, such results are observed in deep water cases.

Higher values of $\gamma_3$ imply higher values of $\gamma_4$ as is expected by the quadratic dependence between $\gamma_4$ and $\gamma_3$ proposed by Malakhov (1978). In this case the theory of Mori and Yasuda (2002) significantly underpredicts crests and overpredicts troughs. The theoretical model of Mori and Yasuda (2002) failed to predict...
accurately the real data. Probably this is due to the deep-water assumptions that were made in the model. The analyzed data were gathered in the coastal zone. Only point 10 and point 11 (Fig. 1) can be referred to as relatively deep-water. Waves are subjected to shoaling, transformation and increasing non-linearity while propagating to the shore. The spectrum is getting broader in contrast to the model assumptions. The mean value of the coefficient of skewness for the measured data is equal to 0.5045; the mean asymmetry is 0.5715 while the model was checked for consistency with field data for small coefficients of skewness ($\gamma_3 \ll 1$), although for considerably high values of the coefficient of kurtosis ($\gamma_4 \leq 1$).

Fig. 13 is related to wave heights. The Rayleigh distribution is found to give a good quantitative representation for small wave heights and larger depths. Deviation from the linear theory is observed over the entire range of wave heights at the shallowest points (Fig. 13f): the smallest wave heights are underpredicted and highest—overpredicted by the Rayleigh fit. At relatively deep water (Fig. 13a) the Edgeworth–Rayleigh form of Mori and Yasuda (2002) describes best the empirical probabilities. In the shallowest case (Fig.
13f), the small waves are better represented by Ochi (1998), because it gives higher probability of occurrence of a certain height value as compared to the linear theory. However, the general tendency is that neither of the proposed distributions can be used to describe well high waves. They are usually overpredicted by the Rayleigh distribution, which complies with the majority of previously reported results (e.g. Forristall, 1978, etc.). Nevertheless, there are cases, when the effect is reversed. For example, the Rayleigh distribution falls under field observations as is demonstrated in Fig. 13c,d and e. The possible reason can be attributed to the non-linear mechanisms, working in the coastal zone and basically to the process of wave breaking. It must be noted that all data have been processed in the same manner, without directly taking into account the effect of breaking and the increasing number of broken waves in the records shoreward.

The visual observations show that there are generally two regions of wave breaking. The highest waves start to break at greatest distances from the shore. Thus, the first breaking spans from about 225 to 185 m offshore, (see Fig. 1, Table 1) when the bottom effects become significant. The second zone is closely to the shoreline at about 65–85 m offshore. As a result of wave breaking, the tail of the wave height distribution is truncated and then Rayleigh overestimates the experimental data. However, in the transitional zone (185–85 m) the broken waves form a new wave system together with still unbroken waves. This is the zone
Fig. 12. Exceedance probability of wave troughs for various water depths: (a) $h = 11.00$ m, $\omega_h = 1.1371$; (b) $h = 4.50$ m, $\omega_h = 0.7273$; (c) $h = 3.20$ m, $\omega_h = 0.6133$; (d) $h = 2.60$ m, $\omega_h = 0.4738$; (e) $h = 1.30$ m, $\omega_h = 0.3350$; (f) $h = 1.60$ m, $\omega_h = 0.3717$. 

Z. Cherneva et al. / Coastal Engineering 52 (2005) 599–615
Fig. 13. Exceedance probability of wave heights, for various water depths: (a) $h=11.00$ m, $\omega_h=1.1371$; (b) $h=4.50$ m, $\omega_h=0.7273$; (c) $h=3.80$ m, $\omega_h=0.6683$; (d) $h=3.20$ m, $\omega_h=0.6133$; (e) $h=2.60$ m, $\omega_h=0.4738$; (f) $h=1.30$ m, $\omega_h=0.3350$. 
where the effects of non-linear interactions are comparable with wave dispersion. Due to shoaling an increase in the wave height is observed. In this case, the results from investigations show enhanced probability of occurrence as compared to the Rayleigh one.

In his theory Ochi (1998) takes into account that envelopes of crests and troughs differ from each other due to the vertical asymmetry of the wave profile and describes them by two distinct distributions. The present results demonstrate that the empirical distributions of ordinates and troughs are well approximated by his theory. Nevertheless, the agreement between theory and experiment for crests and heights is not satisfactory. It could be explained with the increased number of broken waves to the shore. The wave shoaling and breaking processes strongly affect the experimental distribution and contribute to the discrepancy between theory and data.

The theoretical model of Mori and Yasuda (2002) is not capable of accurate prediction of the field data due to the assumption of symmetry between crests and troughs that implies the wave height to be twice the wave amplitude. This condition is not valid in the shallow waters as was shown in Fig. 2. Nevertheless, the distribution is seen to represent well wave heights in deep water (Fig. 13a).

For very large values of the coefficient of skewness and coefficient of kurtosis the theoretical models proposed by Al-Humoud et al. (2002) and Mori and Yasuda (2002) predict negative values for the probability function of crests and troughs. This makes the distribution functions not applicable for these shallow water conditions.

4. Conclusions

In the present work the experimental distributions of wave crests, troughs and heights were investigated. It was demonstrated that the non-dimensional ratio $H_{ct}/H_{tr}$ grows with the decrease in water depth, implying that non-linear effects become stronger. This fact was quantitatively described by the dependence of the coefficients of skewness and kurtosis on $\omega_h$ and two empirical approximations were proposed for this relation.

The theoretical distributions of wave parameters suggested by Ochi (1998), Al-Humoud et al. (2002) and Mori and Yasuda (2002) were compared with field data. It was shown that the empirical probabilities of occurrence of large crests are underestimated by all three models, although the small crests are well described by the distribution of Al-Humoud et al. (2002). The troughs are well described by the theory of Ochi (1998) for $0<\gamma_3<0.9$. For negative values of $\gamma_3$ the trough data are in good agreement with Al-Humoud et al. (2002).

The theoretical model of Mori and Yasuda (2002) is not capable of reproducing field data due to the assumption that the wave height equals twice the wave amplitude. For the shallow waters of this coastal zone the vertical asymmetry of the wind wave profile is too pronounced.

The large values of the coefficients of skewness and kurtosis result in negative values for the crests and troughs probability functions suggested by Al-Humoud et al. (2002) and Mori and Yasuda (2002). It makes the aforementioned distributions not applicable for shallow water conditions.

In some cases, the Rayleigh distribution underestimates the observed wave heights. This result contradicts to the former reports on that issue, where the linear theory is seen to overestimate the field data. A plausible explanation is the wave transformation and increased non-linear effects in the coastal zone, including wave breaking.

Considering the parameters $\omega_h$ and $U_r$, it is seen that $\omega_h$ decreases shoreward while $U_r$ increases. It implies that the transformation in the wave field and hence the form of the distribution of wave elements is increasingly influenced by the non-linear interactions in the coastal zone. The problem needs further clarification, especially as regards the connection between the Ursell parameter and its effect on the form of the distribution.

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